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NONLINEAR ANALYSIS OF RECTANGULAR LAMINATED DECKS PLATES USING LARGE DEFLECTION THEORY

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Abstract: Dynamic Relaxation (DR) method is presented for the geometrically nonlinear laterally loaded, rectangular laminated plates. The analysis uses the Mindlin plate theory which accounts for transverse shear deformation. A computer program has been compiled. The convergence and accuracy of the DR solutions for elastic large deflection response are established by comparison with various exact and approximate solutions. New numerical results are generated for uniformly loaded square laminated plates which serve to quantify the effects of shear deformation, length to thickness ratio, number of layers, material anisotropy and fiber orientation.

It was found that linear analysis seriously over predicts deflection of plates. The shear deflection depends greatly on a number of factors such as length to thickness ratio, degree of anisotropy and number of layers. As the degree of anisotropy increases, the plate becomes stiffer and when it is greater than a critical value, the deflection becomes virtually independent on the degree of anisotropy. It was also found that deflection of plates depends on the angle of orientation of individual plies and the size of load applied.

Notations

a, b plate side lengths in x and y directions respectively

$A_{ij}(i, j = 1,2,6)$ Plate in plane stiffness

A_{44}, A_{55} Plate transverse shear stiffness

$D_{ij}(i, j = 1,2,6)$ Plate flexural stiffness

$\varepsilon_x^{\circ}\ \varepsilon_y^{\circ}\ \varepsilon_{xy}^{\circ}$ Mid – plane direct and shear strains

$\varepsilon_{xz}^{\circ}, \varepsilon_{yz}^{\circ}$ Mid – plane transverse shear strains

E_x, E_y, G_{xy} In – plane elastic longitudinal, transverse and shear moduli

G_{xz}, G_{yz} Transverse shear moduli in the x – z and y – z planes respectively

M_x, M_y, M_{xy} Stress couples

$\overline{M}_x, \overline{M}_y, \overline{M}_{xy}$ Dimensionless stress couples

N_x, N_y, N_{xy} Stress resultants

$\overline{N}_x, \overline{N}_y, \overline{N}_{xy}$ Dimensionless stress resultants

$\overline{q}$ Dimensionless transverse pressure

Q_x, Q_y Transverse shear resultants

u, v In – plane displacements.

w Deflections

$\overline{w}$ Dimensionless deflection

x, y, z Cartesian co – ordinates

ϕ, ψ Rotations of the normal to the plate mid – plane

ν_{xy} Poisson’s ratio

$\chi^{\circ}_x, \chi^{\circ}_y, \chi^{\circ}_{xz}$ Curvature and twist components of plate mid – plane

1 INTRODUCTION

The three dimensional theories of laminates in which each layer is treated as homogeneous anisotropic medium (Reddy [1]) are intractable as the number of layers becomes moderately large. Thus, a simple two dimensional theory of plates that accurately describes the global behaviour of laminated plates seems to be a compromise between accuracy and ease of analysis. Putcha and Reddy [2] classified the two dimensional analyses of laminated composite plates into two categories: (1) the classical lamination theory, and (2) shear deformation theories. In both theories it is assumed that the laminate is in a state of plane stress, the individual lamina is linearly elastic, and there is perfect bonding between layers. The classical laminated plate theory (CLPT) was the first theory formulated for the analysis of laminated plates by Reissner and Stavsky [3], in which the Kirchhoff-love assumption that normal to the mid surface before deformation remains straight and normal to the mid surface after deformation is used, but it is not adequate for the flexural analysis of moderately thick laminates. However, it gives reasonably accurate results for many engineering problems i.e. thin composite plates, as verified by Srinivas and Rao [4], Reissner and Stavsky [3], Hui – Shen Shen [5], Ji Fan He and Shunag – Wang Zheng [6]. This theory ignores the transverse shear stress components and models a laminate as an equivalent single layer. The classical laminated plate theory (CLPT) under predicts deflection as proved by Turvey and Osman [7 – 9], and Reddy [1] due to the neglect of transverse shear strains. The errors in deflections are even higher for plates made of advanced filamentary composite materials like graphite epoxy and boron epoxy, whose elastic modulus to shear modulus ratios are very large. However, these composites are susceptible to thickness effects because their effective transverse shear moduli are significantly smaller than the effective elastic modulus along the fiber direction. This effect has been confirmed by Pagano [10], Taner Timarci and Metin Aydogdu [11] who obtained analytical solutions of laminated plates in bending based on the three dimensional theory of elasticity. They proved that classical laminated plated theory (CLPT) becomes of less accuracy as the side to thickness ratio decreases. In particular, the deflection of a plate predicted by CLPT is considerably smaller than the analytical value for side to thickness ratio less than 10. These high ratios of elastic modulus to shear modulus render classical laminate theory as inadequate for the analysis of composite plates.

Many theories which account for the transverse shear and normal stresses are classified according to Phan and Reddy [12] into two major classes on the basis of the assumed fields as: (1) stress based theories, and (2) displacement based theories. The stress based theories are derived from stress fields, which are assumed to vary linearly over the thickness of the plate, and the displacement based theories which are derived from an assumed displacement field. The theory used in the present work comes under the class of displacement based theories. Extensions of these theories which account for higher order variations and applied to laminated plates, can be found in the work of Yang et al. [13], Whitney and pagano [14] and Phan and Reddy [12]. In this theory which is called first order shear deformation theory (FSDT), the transverse planes, which are originally normal and straight to the mid plane of the plate, are assumed to remain straight but not necessarily normal after deformation, and consequently shear correction factors are employed in this theory to adjust the transverse shear stress. Numerous studies involving the application of the first order theory to bending analyses can be found in the works of Reddy [15], Reddy and Chao [16], Prabhu Madabhusi – Raman and Julio F. Davalo [17], and Wang J. et al. [18].

In the present work, a numerical method known as Dynamic Relaxation (DR) coupled with finite differences is used. The DR method was first proposed in 1960th, and then passed through a series of studies to verify its validity by Turvey and Osman [7 – 9], and Rushton [19], Cassell and Hobbs [20], Day [21] and Aalami [22]. In this method, the equations of equilibrium are converted to dynamic equations by adding damping and inertia terms. These are then expressed in finite difference form and the solution is obtained through iterations. The optimum damping coefficient and time increment used to stabilize the solution depend on a number of factors including the properties of the stiffness matrix of the structure, the applied load, the boundary conditions and the size of the mesh used, etc…

2 LARGE DEFLECTION THEORIES

The equilibrium, strain, constitutive equations and boundary conditions are introduced below without derivation.

2.1 Equilibrium equations

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0$$

$$N_x \frac{\partial^2 w}{\partial x^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0 \qquad (1)$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = 0$$

2.2 Strain equations

The large deflection strains of the mid – plane of the plate are as given below:

$$\varepsilon_x^\circ = \frac{\partial u^\circ}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + z\frac{\partial \phi}{\partial x}$$

$$\varepsilon_y^\circ = \frac{\partial v^\circ}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + z\frac{\partial \psi}{\partial y}$$

$$\varepsilon_{xy}^\circ = \frac{\partial u^\circ}{\partial y} + \frac{\partial v^\circ}{\partial x} + \frac{\partial w}{\partial x} - \frac{\partial w}{\partial y} + z\left(\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right) \qquad (2)$$

$$\varepsilon_{xz}^\circ = \frac{\partial w}{\partial y} + \psi$$

$$\varepsilon^{\circ}_{yz} = \frac{\partial w}{\partial x} + \phi$$

2.3 The constitutive equations

The laminate constitutive equations can be represented in the following form:

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon^{\circ}_j \\ \chi^{\circ}_j \end{Bmatrix}$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon^{\circ}_{yz} \\ \varepsilon^{\circ}_{xz} \end{Bmatrix} \tag{3}$$

Where N_i denotes N_x , N_y and N_{xy} and M_i denotes M_x, M_y and M_{xy} . A_{ij}, B_{ij} and D_{ij} $(i, j = 1,2,6)$ are respectively the membrane rigidities, coupling rigidities and flexural rigidities of the plate. χ°_j Denotes $\frac{\partial \phi}{\partial x}, \frac{\partial \psi}{\partial y}$ and $\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}$. A_{44}, A_{45} And A_{55} denote the stiffness Coefficients and are calculated as follows:

$$A_{ij} = \sum_{k-1}^{n} k_i \, k_j \int_{z_k}^{z_{k+1}} c_{ij} \, dz, (i, j = 4,5)$$

Where c_{ij} the stiffness of a lamina is referred to the plate principal axes and $\mathbf{k}_i$, $\mathbf{k}_j$ are the shear correction factors.

2.4 Boundary conditions

Five sets of simply supported boundary conditions are used in this paper, and are denoted as SS1, SS2, SS3, SS4 and SS5 as has been shown in fig (a) below.

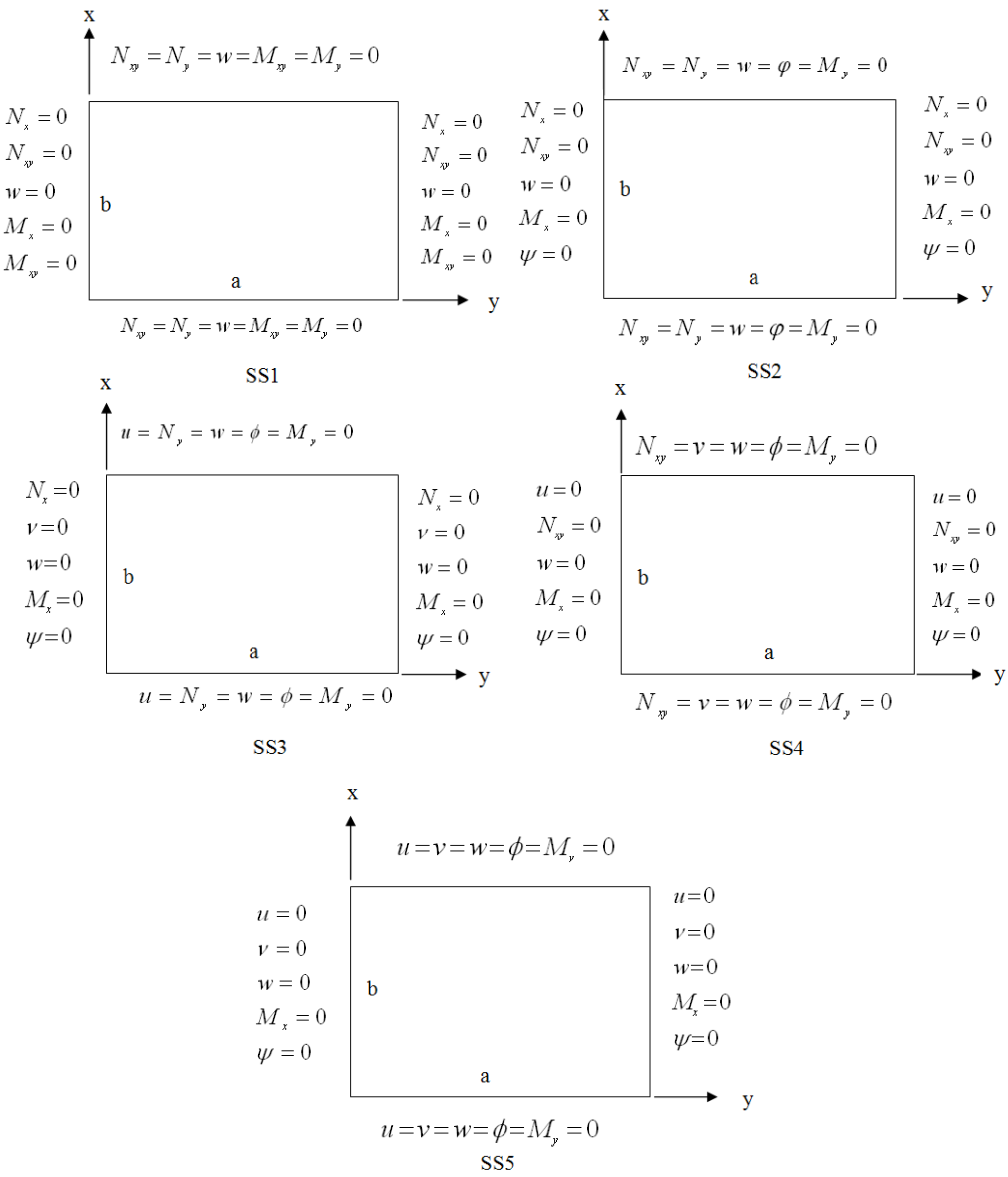

Figure a. Simply supported boundary conditions

3 VERIFICATION OF THE DYNAMIC RELAXATION (DR) METHOD

Table (1) shows deflections, stress resultants and stress couples in simply supported (SS4) isotropic plate. It is apparent that the center deflections, stress couples and stress resultants agree very well with Aalami and Chapman [23].

The mid side stress resultants do not show similar agreement whilst the corner stress resultants show considerable differences. This may be attributed to the type of mesh used in each analysis. A set of thin plate results comparisons presented here with Rushton [19] who employed the DR method coupled with finite differences. The present results for simply supported (SS5) square plates were computed for two thickness ratios and are listed in table (2). In this instant, the present results differ slightly from those found in ref. [19]. Another comparison for simply supported (SS5) square isotropic plates subjected to uniformly distributed loads are shown in table (3) for deflection analysis of thin and moderately thick plates. In this comparison, it is noted that, the center deflection of the present DR analysis, and those of Azizian and Dawe [24] who employed the finite strip method are in fairly good agreement.

A large deflection comparison for orthotropic plates was made with the DR program. The results are compared with DR results of Turvey and Osman [8], Reddy's [25], and Zaghloul et al. [26]. For a thin uniformly loaded square plate made of material I which its properties are stated in table (4) and with simply supported in plane free (SS3) edges. The center deflections are presented in table (5) where DR showed a good agreement with the other three.

A large deflection comparison for laminated plates was made by recomputing Sun and Chin's results [27] for [$90^{\circ}_{4}/0^{\circ}_{4}$] using the DR program and material II which its properties are cited in table (4). The results were obtained for quarter of a plate. The analysis was made for different boundary conditions and the results were shown in tables (6), and (7) as follows: The present DR deflections of two layered antisymmetric cross ply simply supported in plane fixed (SS5) plates are compared with DR results of Turvey and Osman [9] and with Sun and Chin's values for a range of loads as shown in table (6). The good agreement found confirms that for simply supported (SS5) edge conditions, the deflection depends on the direction of the applied load or the arrangement of the layers. Table (7) shows a comparison between the present DR, and DR ref. [9] results, which are approximately identical.

The comparisons made between DR and alternative techniques show a good agreement and hence the present DR large deflection program using uniform finite difference meshes can be employed with confidence in the analysis of moderately thick and thin flat isotropic, orthotropic or laminated plates under uniform loads. The program can be used with the same confidence to generate small deflection results.

Table 1. Comparison of present DR, Aalami and Chapman's [23] large deflection results for simply supported (SS4) square isotropic plate subjected to uniform pressure

$\bar{q}$	S	$\bar{w}_c$	$\bar{M}_x(1)$ $\bar{M}_y(1)$	$\bar{N}_x(1)$ $\bar{N}_y(1)$	$\bar{N}_x(2)$ $\bar{N}_y(3)$	$\bar{N}_x(3)$ $\bar{N}_y(2)$	$\bar{N}_x(4)$ $\bar{N}_y(4)$
20.8	1 2	0.5994 0.6094	0.6077 0.6234	1.0775 1.0714	0.2423 0.2097	1.1411 1.1172	0.1648 0.2225
41.6	1 2	0.8613 0.8783	0.8418 0.8562	2.2435 2.2711	0.5405 0.4808	2.4122 2.4084	0.3177 0.4551
63.7	1 2	1.0434 1.0572	0.9930 1.0114	3.3151 3.3700	0.8393 0.7564	3.6014 3.6172	0.4380 0.6538

S (1): present DR results
S (2): ref. [23] results

$$(1)\, x = y = \frac{1}{2}a\,, z = 0\,; (2)\, x = \frac{1}{2}a\,, y = z = 0\,; (3)\, x = 0\,, y = \frac{1}{2}a\,, z = 0\,; (4)\, x = y = z = 0$$

Table 2. Comparison of present DR, and Rushton's [19] large deflection results for simply supported (SS5) square thin isotropic plate subjected to uniform pressure

$\bar{q}$	S	$\bar{w}_c$	$\bar{\sigma}_1(1)$
29.3	1 2	0.7249 0.7310	5.9580 6.2500
91.6	1 2	1.2147 1.2200	11.3249 11.4300
293.0	1 2	1.8755 1.8700	20.752 20.820

S (1): present DR results
S (2): ref. [19] results

$$(1)\, x = y = \frac{1}{2}a\,, z = \frac{1}{2}h$$

Table 3. Comparison of the present DR, and Azizian and Dawe's [24] large deflection results for moderately thick shear deformable simply supported (SS5) square isotropic plates subjected to uniform pressure

$\bar{q}$	S	$\bar{w}_c$
0.92	**1**	**0.04106**
	2	**0.04105**
4.6	**1**	**0.19493**
	2	**0.19503**
6.9	**1**	**0.27718**
	2	**0.27760**

S (1): present DR results
S (2): Azizian and Dawe [**24**] results.

Table 4. Material properties used in the orthotropic and laminated plate comparison analysis.

Material	E_x / E_y	G_{xy} / E_y	G_{xz} / E_y	G_{yz} / E_y	ν_{xy}	$SCF\left(k_4^2 = k_5^2\right)$
I	**2.345**	**0.289**	**0.289**	**0.289**	**0.32**	$5/6$
II	**14.3**	**0.5**	**0.5**	**0.5**	**0.3**	$5/6$

Table 5. Comparison of present DR, DR results of ref. [8], finite element results ref. [25] and experimental results ref. [26] for a uniformly loaded simply supported (SS3) square orthotropic plate made of material I

$\bar{q}$	$\bar{w}_c(1)$	$\bar{w}_c(2)$	$\bar{w}_c(3)$	$\bar{w}_c(4)$
17.9	**0.5859**	**0.5858**	**0.58**	**0.58**
53.6	**1.2710**	**1.2710**	**1.30**	**1.34**
71.5	**1.4977**	**1.4977**	**1.56**	**1.59**

(1): present DR results

(2): DR results of ref. [8].

(3): Reddy's finite element results [25].

(4): Zaghloul's and Kennedy's [26] experimental results as read from graph.

Table 6. Deflection of the center of a two layered antisymmetric cross ply simply supported in plane fixed (SS5) strip under uniform pressure.

$\overline{q}$	S	$\overline{w}_1[0^\circ/90^\circ]$	$\overline{w}_2[90^\circ/0^\circ]$	$\overline{w}_\circ(B_{ij}=0)$	$\%(1)$	$\%(2)$	$\%(3)$
18	1 2 3	0.6851 0.6824 0.6800	0.2516 0.2544 0.2600	0.2961	131.4 130.5	- 15.0 - 14.1	172.3 168.2
36	1 2 3	0.8587 0.8561 0.8400	0.3772 0.3822 0.3900	0.4565	88.1 87.5	- 17.4 - 16.3	127.7 124.0
72	1 2 3	1.0453 1.0443 1.0400	0.5387 0.5472 0.5500	0.6491	61.0 60.9	- 17.0 - 15.7	94.0 90.8

S (1): present DR results

S (2): DR results ref. [9].

S (3): Values determined from sun and chin's results ref. [27].

(1): $100\times(\overline{w}_1-\overline{w}_\circ)/\overline{w}_\circ$

(2): $100\times(\overline{w}_2-\overline{w}_\circ)/\overline{w}_\circ$

(3): $100\times(\overline{w}_1-\overline{w}_2)/\overline{w}_2$

Table 7. Center deflection of two layered antisymmetric cross ply simply supported in plane free (SS1) plate under uniform pressure and with different aspect ratios $(\mathrm{h/a}=0.01;\overline{\mathrm{q}}=18)$.

b/a	S	$\overline{w}_1[0^\circ/90^\circ]$	$\overline{w}_2[90^\circ/0^\circ]$	$\overline{w}_\circ(B_{ij}=0)$	$\%(1)$	$\%(2)$	$\%(3)$
5.0	1 2	0.8691 0.8683	0.8718 0.8709	0.3764 0.3764	130.9 129.1	131.6 130.2	- 0.3 - 0.3
4.0	1 2	0.8708 0.8708	0.8758 0.8557	0.3801 0.3801	129.1 129.1	129.1 130.4	- 0.6 - 0.6
3.0	1 2	0.8591 0.8593	0.8677 0.8678	0.3883 0.3883	121.2 121.3	123.5 123.5	- 1.0 - 1.0

2.5	1 2	0.8325 0.8328	0.8422 0.8424	0.3907 0.3907	113.1 113.2	115.6 115.6	- 1.15 - 1.1
2.0	1 2	0.7707 0.7712	0.7796 0.7799	0.3807 0.3807	102.4 102.6	104.8 104.9	- 1.14 - 1.1
1.75	1 2	0.7173 0.7169	0.7248 0.7251	0.3640 0.3640	97.0 97.0	99.1 99.2	- 1.0 - 1.1
1.5	1 2	0.6407 0.6407	0.6460 0.6455	0.3335 0.3325	92.1 92.7	93.7 94.1	- 0.82 - 0.70
1.25	1 2	0.5324 0.5325	0.5346 0.5347	0.2781 0.2782	91.4 91.4	92.2 92.2	- 0.4 - 0.4
1.0	1 2	0.3797 0.3796	0.3797 0.3796	0.1946 0.1949	95.1 94.8	95.1 94.8	0.0 0.0

S (1): present DR results

S (2): DR results Ref. [9].

(1): $100 \times \left(\overline{w}_1 - \overline{w}_\circ\right) / \overline{w}_\circ$

(2): $100 \times \left(\overline{w}_2 - \overline{w}_\circ\right) / \overline{w}_\circ$

(3): $100 \times \left(\overline{w}_1 - \overline{w}_2\right) / \overline{w}_2$

4 New Numerical Results

It was decided to undertake some study cases and generate results for uniformly loaded laminated rectangular plates. The plates were assumed to be simply supported on all edges. The effects of transverse shear deformation, material anisotropy, orientation, and coupling between stretching and bending on the deflections of laminated plates are investigated. The material chosen has the following properties:

$$E_x = 137.9\,kN/mm^2, E_y = 9.653kN/mm^2, G_{xy} = 4.8265\,kN/mm^2, \nu_{xy} = 0.3$$

It is assumed that $G_{xy} = G_{xz} = G_{yz}$.

Effect of load

The variations of the center deflections, $\vec{w}_c$ with load, $\vec{q}$ for thin and thick isotropic plates of simply supported in plane fixed (SS5) condition are given in table (8), and fig. (1). It is observed that, the center deflections of thin and thick plates increase with the applied load, and that the deflections of thick plates are greater than those of thin plates under the same loading conditions. The difference in linear deflection is due to shear deformation effects which are significant in thick plates. Whereas, the nonlinear difference of thin and thick plates, which are approximately coincident, implies that the shear deformation effect vanishes as the load is increased.

Effect of length to thickness ratio

Table(9) and fig.(2) contain numerical results and plots of center deflection versus length to thickness ratio of 4 – layered antisymmetric cross ply and angle ply square plates under lateral load $(\bar{q} = 1.0)$ for simply supported (SS1) boundary condition. The maximum percentage difference in deflections for a range of length to thickness ratio between 10 and 100 fluctuates between 35% for cross ply laminate and 73.3% for angle ply laminate as the length to thickness ratio increases to a value of a/h = 40.0, and then becomes fairly constant. It is evident that shear deformation effect is significant for a/h < 40.0. It is obvious that shear deformation reduces as the length to thickness ratio increases.

Effect of number of layers

Fig. (3) Shows a plot of the maximum deflection of a simply supported (SS5) antisymmetric cross ply $\left[(0^\circ/90^\circ)_n\right]$ $(n = 1,2,3,4,8)$ square plates under uniformly distributed load of a moderately thick plate (h/a = 0.1). The numerical results are given in table (10). Two, four, six, eight, and sixteen layered laminates are considered. The results show that as the number of layers increases, the plate becomes stiffer and deflection becomes smaller. This is mainly due to the existence of coupling between bending and stretching which generally increases the stiffness of the plate as the number of layers is increased. When the number of layers exceeds 8, the deflection becomes independent on the number of layers. This is because the effect of coupling between bending and stretching does not change as the number of layers increases beyond 8 layers.

Effect of material anisotropy

The exact maximum deflections of simply supported (SS5) four layered symmetric cross ply and angle ply laminates are compared in table (11) and fig. (4) for various degrees of anisotropy. It is observed that, when the degree of anisotropy is small the deflection is large. As the degree of the anisotropy increases, the plate becomes stiffer. This may be attributed to the shear deformation effect which increases as the material anisotropy decreases. When the degree of anisotropy becomes greater than 40.0, the deflection becomes approximately independent on the degree of anisotropy. This is due to the diminishing of the shear deformation effects and the dominance of bending effects.

Effect of fiber orientation

The variation of the maximum deflection, $\vec{w}_c$ with fiber orientation of a square laminated moderately thick plate is shown in table (12) and fig. (5). Four simply supported boundary conditions SS2, SS3, SS4 and SS5 are considered in this case. The nonlinear curves of SS2 and SS3 conditions show minimum deflection at $\theta = 45^o$. However, this trend is different for a plate under SS4 and SS5 conditions in which the nonlinear deflection increases with θ. This is due to the in plane fixed edges in the latter case. Another set of results showing the variation of center deflections, $\vec{w}_c$ with load $\bar{q}$ for a range of orientations is given in table (13) and fig. (6). They show the variations in the center deflection of thick laminates with wide range of loads for a simply supported (SS4), 4 layered antisymmetric square plate of orientation [θ°, $-\theta^\circ, \theta^\circ, -\theta^\circ$]. It is noticed from fig. (6) that the deflection of thick laminates increases with the applied load as the angle of orientation is decreased (i.e. from 45° to 0°) to a point where $60 < \bar{q} \le 70$ and then increases as the angle of orientation is increased beyond that point. This results in the inflection of the deflection curves at a point where $60 < \bar{q} \le 70$. This behaviour is caused by coupling between bending and stretching which arises as the angle of orientation increases.

Table 8. Variation of central deflection with load, of thin and thick isotropic plates of simply supported (SS5) condition

$\bar{q}$	S	$\bar{w}_c$ h /a = 0.02	$\bar{w}_c$ h /a = 0. 2
20	1	0.8856	1.0635
	2	0.5846	0.6159
60	1	2.6562	3.1906
	2	1.0138	1.0262
100	1	4.4270	5.3177
	2	1.2527	1.2573
140	1	6.1979	7.4448
	2	1.4275	1.4279
180	1	7.9685	9.578
	2	1.5685	1.5662

200	1 2	8.8541 1.6306	10.6354 1.6274

S (1): Linear
S (2): Nonlinear

Table 9. A comparison of dimensionless center deflections VS side to thickness ratio of a four layered antisymmetric cross ply and angle ply simply supported (SS1) square laminates under uniform unit lateral load

a/h	$\overline{w}_c$	
	$[0^\circ/90^\circ/0^\circ/90^\circ]$	$[45^\circ/-45^\circ/45^\circ/-45^\circ]$
10	0.0148	0.0115
20	0.0134	0.0097
30	0.0132	0.0094
40	0.0131	0.0092
80	0.0130	0.0091
100	0.0130	0.0091

Table 10. Number of layers effect on a simply supported (SS5) antisymmetric cross ply $[(0^\circ/90^\circ)_n]$ square thick plate under uniformly distributed loads (h/a = 0.1)

$\overline{q}$	$\overline{w}_c$				
	$[0^\circ/90^\circ]$	$[0^\circ/90^\circ]_2$	$[0^\circ/90^\circ]_3$	$[0^\circ/90^\circ]_4$	$[0^\circ/90^\circ]_8$
20	0.2953	0.2278	0.2250	0.2241	0.2232
60	0.5287	0.4807	0.4458	0.4742	0.4727
100	0.6725	0.6258	0.6201	0.6182	0.6165
140	0.7791	0.7304	0.7242	0.7221	0.7202
180	0.8639	0.8136	0.8071	0.8049	0.8029
200	0.9009	0.8500	0.8433	0.8411	0.8490

Subscripted values 2, 3, 4, and 8: No. of the arrangement of a two of layered laminate.

Table 11. Effect of material anisotropy on the dimensionless center deflection of a four layered symmetric cross ply and angle ply simply supported thick laminates (SS5) under uniform lateral load.

E_x / E_y	$\overline{w}_c$	
	[0° / 90° / 0° / 90°]	**[45° / - 45° / 45° / 45°]**
2	1.1114	1.1114
8	0.7610	0.7466
14	0.6218	0.5962
20	0.5410	0.5098
30	0.4589	0.4242
40	0.4076	0.3724
50	0.3718	0.3374

Table 12. Effects of fiber orientation θ on the deflection of a simply supported thick square plate

θ	$\overline{w}_c$			
	SS2	**SS3**	**SS4**	**SS4**
0	**1.3706**	**1.2346**	**0.6511**	**0.6513**
15	**1.3359**	**1.1769**	**0.7434**	**0.6713**
30	**1.2438**	**1.0321**	**0.8173**	**0.7101**
45	**1.1815**	**0.9056**	**0.8049**	**0.7267**
60	**1.2438**	**1.0321**	**0.8173**	**0.7101**
75	**1.3359**	**1.1769**	**0.7434**	**0.6713**
90	**1.3706**	**1.2346**	**0.6511**	**0.6513**

Table 13. Variation of central deflection $\overline{w}_c$ with a high pressure range $\overline{q}$ of a simply supported (SS4) four layered antisymmetric square plate of the arrangement $\left[\theta^{\circ}/-\theta^{\circ}/\theta^{\circ}/-\theta^{\circ}\right]$ with different orientations (h/a = 0.2).

$\overline{q}$	$\overline{w}_c$			
	$\theta = 0\, or\, 90^{\circ}$	$\theta = 15^{\circ}\, or\, 75^{\circ}$	$\theta = 30^{\circ}\, or\, 60^{\circ}$	$\theta = 45^{\circ}$
20	**0.2922**	**0.2799**	**0.2568**	**0.2466**
40	**0.4268**	**0.4209**	**0.4098**	**0.4039**
60	**0.5150**	**0.5141**	**0.5142**	**0.5135**
80	**0.5826**	**0.5853**	**0.5943**	**0.5948**
100	**0. 6382**	**0. 6438**	**0. 6603**	**0. 6685**
120	**0. 6859**	**0. 6940**	**0. 7169**	**0. 7286**
160	**0. 7660**	**0. 7779**	**0. 8114**	**0. 8292**
200	**0.8326**	**0.8475**	**0.8896**	**0.9124**

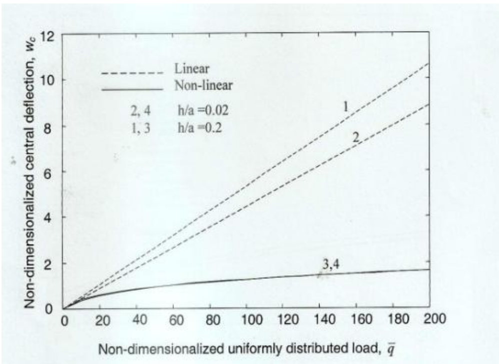

Fig. 1 Variation of central deflection with load of thin and thick simply supported (SS5) square isotropic plate.

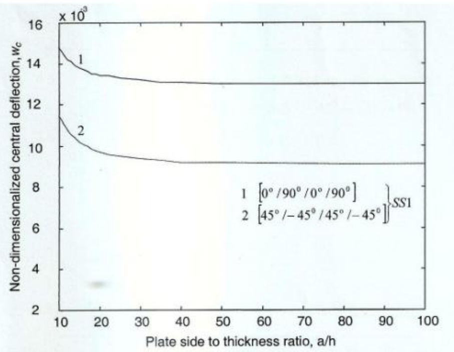

Fig. 2 A comparison of the non-dimensionalized centre deflections versus side to thickness ratio of anti-symmetric cross-ply and angle-ply square laminates under uniform lateral load ($\overline{q}$ =1.0).

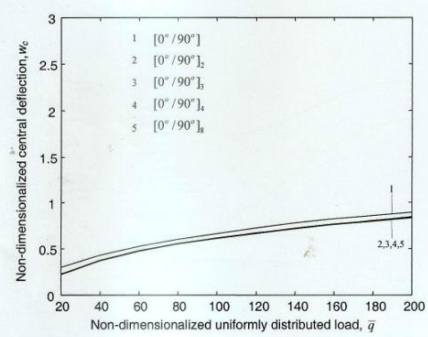

Fig. 3 Number of layers effect on a simply supported (SS5) antisymmetric cross-ply [$(0^o/90^o)_n$] square plate under uniformly distributed loads ($h/a = 0.1$).

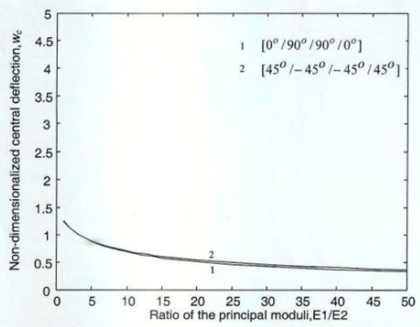

Fig. 4 Effect of material anisotropy on the non-dimensionalized centre deflections of a four layered symmetric cross-ply and angle-ply simply supported laminates (SS5) under uniform lateral load ($\bar{q}$ = 100.0, $h/a = 0.1$).

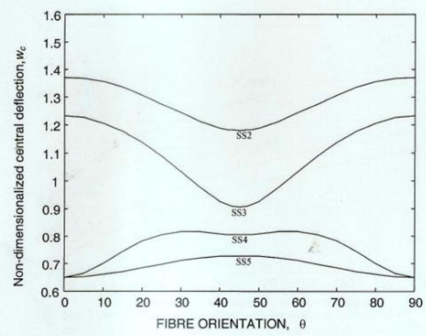

Fig. 5 Effects of fiber orientation, θ on the deflection of a simply supported square plate ($\bar{q}$ = 120.0, $h/a = 0.1$).

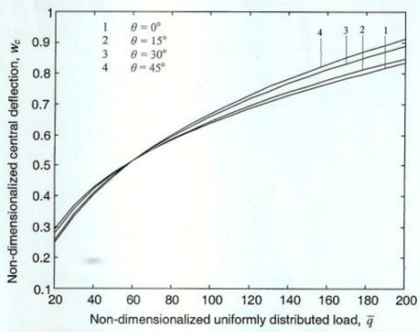

Fig. 6 Variation of central deflection with pressure of simply supported (SS4) antisymmetric square plate of the arrangement [θ°/-θ° /θ°/ -θ°] with different orientations (h/a =0.2).

6 CONCLUSIONS

A Dynamic relaxation (DR) program based on finite differences has been developed for large deflection analysis of rectangular laminated plates using first order shear deformation theory (FSDT). The plate, which is assumed to consist of a number of orthotropic layers, is replaced by a single anisotropic layer and the displacements are assumed linear through the thickness of the plate. A series of numerical comparisons have been undertaken to demonstrate the accuracy of the DR program. Finally, a series of new results for uniformly loaded thin, moderately thick, and thick plates with simply supported edges have been presented. These results show the following:

1. The linear theory seriously over predicts the deflection of plates.
2. The deformations of a plate are dependent on bending and extension in the nonlinear theory, whereas they are dependent on bending alone in the linear theory.
3. Convergence of the DR solution depends on several factors including boundary conditions, meshes size, fictitious densities and applied load.

4. Deflection is greatly dependent on plate length to thickness ratio (a/h) at small loads, and it becomes almost independent on that when the load is large.
5. As the number of layers in a plate increases, the plate becomes increasingly stiffer.
6. As the degree of anisotropy increases, the plate becomes stiffer and when it is greater than 40.0, the deflection becomes virtually independent on the degree of anisotropy.
7. Deflection of plates depends on the angle of orientation of individual plies. An increase of angle of orientation results in a decrease in the deflection at small loads and an increase in deflection at large loads.

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